NEAR CIRCULAR ORBITS :
A SEMI-ANALYTICAL METHOD WHICH PERMITS TO CALCULATE ORBITS OVER SEVERAL YEARS

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ABSTRACT *– We have developed a semi-analytical method which enables us to calculate orbits over large time scales. This method, formulated in keplerian elements since ten years, is now also adapted to near circular orbits, like those of the GRACE mission. Through our concept of centered motion, this method affords to use an integration step 100 to 1000 times higher than the step used in merely numerical methods. We can thus generate and study orbits, near circular or not, focusing on the only long period effects.*

KEYWORDS: semi-analytical integration, orbital elements, near circular orbits, mission analysis.

INTRODUCTION

This talk takes place in the context of dynamical studies lead in Observatoire de la Côte d'Azur, France. It is focused on the fact that analytical and numerical tools we developped are well adapted to the calculation of low altitude orbits, all the more that the entire algorithm has been reinvestigated in non singular elements for eccentricity.

In opposition to a description of satellites' trajectories realized through positions and velocities, a description through orbital elements makes it possible to link much more easily the time evolution of characteristics of orbits with perturbative forces (gravitational field, thrusts...). Focusing on mission analysis, this time evolution has to be described over long time scales. Our approach of centered motion, and of averaged elements, affords that kind of description.

Our first purpose consists in presenting the principles of our method to calculate orbits, through the notion of centered motion. The method is essentially based on the elimination of the mean anomaly from the equations of motion, through a transformation of the hamiltonians for gravity potentials or directly from the expression of the other forces. As a result, comparing with other techniques of averaging (merely numerical for example), our concept of centered motion is much more accurate in defining and realizing properly the separation between short and long period terms.

Our second purpose is to show the exhaustiveness of our method.

The concept of centered motion, already developed in keplerian elements since ten years, is now also adapted to near circular orbits, and in particular for all low altitude satellites. The ability of calculating near circular orbits is very important: as well as many geodetic satellites, all low altitude satellites have a circular orbit, as well as constellations such as GPS or GALILEO, let alone future missions around other planets or satellites.
On the other hand, even if this method is based on analytical transformations, all perturbations suffered by satellites are taken into account: terrestrial gravitational field, including high degrees, gravitational field generated by a third body, terrestrial and oceanic tides, atmospheric drag, radiation pressure...

Our final purpose is to show how this method can be managed through two concrete examples, based on an extrapolation of near circular orbits, those of CHAMP and GALILEO.
The example of the orbit of CHAMP shows the level of accuracy reached with this method: effects with an amplitude about a few centimeters can be managed.
The example of the future constellation GALILEO shows how this method can be applied to analysis missions.
In both cases, times of calculation between our method and a merely numerical method are compared.

1. OUR CONCEPT OF CENTERED MOTION

1.1.The notion of centered motion

We call centered motion, the motion described by 6 parameters from which we have removed short periodic variations. Thus the centered motion is very closed from what is usually called mean motion. The difference comes solely from the fact that centered variables must still contain all long periodic variations whereas a small part of theses variations are often lost in mean variables ([Métris, 1991]).

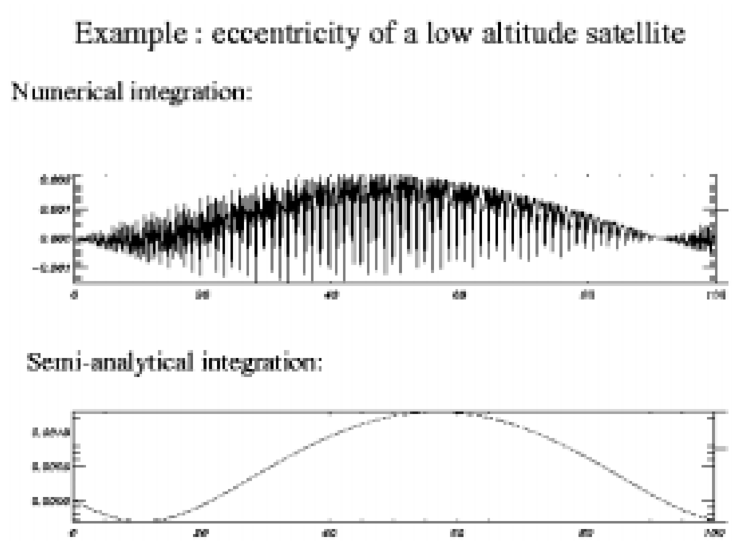
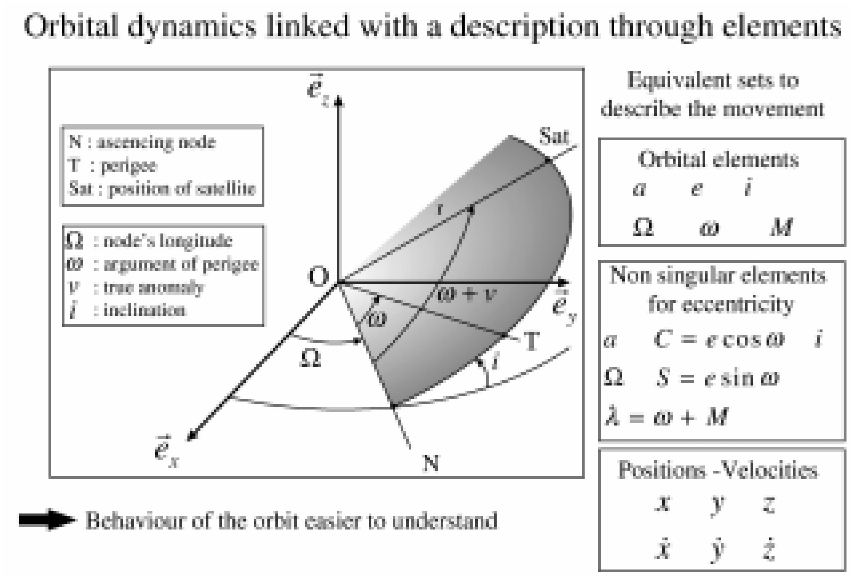


Fig. 1. A motion numerically and semi-analytically integrated

1.2. Orbital elements and non singular elements for eccentricity



The exact separation between short and long period variations is linked with the type of variables used to parametrize the movement.

The orbit of a satellite is normally described with six elliptic elements : the semi-major axis *a*, the eccentricity *e*, the inclination *i*, the longitude of the ascending node Ω, the argument of perigee ω the mean anomaly *M*.

Moreover, it is well known that the transformation $C = e\cos\omega$, $S = e\sin\omega$ and $\lambda = \omega + M$ ensures a suitable description of near circular orbits.

1.3. Principle of our method

The method is based on the elimination of the mean anomaly from the equations of motion, through a transformation of the hamiltonians for gravity potentials. The algorithm is given in [Deprit, 1969]. It leads to what are usely named « mean elements ». In order to get centered elements, we have to add a second analytic step ([Métris and Exertier, 1995]). As far as non gravitational forces are concerned, specific transformations have to be done, directly from their.

After this analytical work performed once for all (see [Claes,1988]), the averaged equations are integrated numerically for a given satellite.

2. EXHAUSTIVENESS OF OUR METHOD

2.1. Classes of orbits

The method has already been applied to manage orbits of geodetic satellites such as STARLETTE and LAGEOS, whose eccentricity is not too small.

But, many satellites have a circular orbit: all low altitude satellites -including space debris-, many geodetic satellites, navigation constellations such as GPS or Galileo, let alone future missions around other planets or satellites.

To remove entirely the singularity of eccentricity, a mere projection from a set of orbital elements to another one is not sufficient: all equations of dynamics have to be reformulated, from Lagrange's equations to the expression of perturbative forces themselves. The aim is therefore to obtain a second member of the averaged differential equations expressed with the same dynamic variables as the left member, i.e. in both classes or orbits :

$$\frac{dE'}{dt} = f(E', \varepsilon_k, t) \tag{1.}$$

where ε_k denotes a set of geodynamical parameters, *t* the time, and *E'* the set of orbital elements chosen, deduced from *E* through the principle of averaging described in section 1. *E* can be equal to $(a, e, i, \Omega, \varpi, M)$ or to $(a, C, i, \Omega, S, \lambda)$.

2.2. Classes of perturbations

All perturbations suffered by a satellite can be taken into account.

2.2.1. Gravitational averaged perturbations

Gravitational perturbations are formulated through an hamiltonian formulation. For example, the averaged hamiltonian associated to the long period effects due to J_2 can be written such as :

$$\mathcal{H}_0^1 = -J_2 \frac{R_0^2 \mu^4}{L^6} \frac{1}{(1-e^2)^{\frac{3}{2}}} \left(-\frac{1}{2} + \frac{3}{4}\sin^2 i\right) \tag{2.}$$

in a closed form, or :

$$\mathcal{H}_0^1 = -J_2 \frac{R_0^2 \mu^4}{L^6} \left(\frac{1}{2} - \frac{3}{4}\sin^2 i + \frac{3}{4}e^2 - \frac{9}{8}e^2 \sin^2 i + \frac{15}{16}e^4 - \frac{45}{32}e^4 \sin^2 i\right) \tag{3.}$$

expanded in powers of eccentricity up to degree 4.

R_0 denotes the equatorial radius of the Earth, μ the product of the mass of the Earth and of the gravitational constant, and $L = \sqrt{\mu a}$.

From a numerical point of view, it is more adapted to use closed form or expanded form, whether integration variables are the classical orbital elements, or the non singular elements for eccentricity.

Expressions of the hamiltonians have been calculated for the following forces:

1. Gravitational field of the Earth, from J_2 to J_{50}

2. Gravitational field due to a third body (Moon, Sun, other planets)
3. Terrestrial tides

These expressions are used to integrate numerically the averaged differential system, through averaged Lagrange equations which are the classical Lagrange equations (in that case, only the perturbation is averaged).

2.2.2. Non gravitational averaged perturbations

Non gravitational averaged perturbations are used through the following averaged Gauss equations, expressed either in classical elements or in not singular elements for eccentricity.

The perturbative force is described through its components *(R,T,N)* in a orbital frame oriented by the radial vector. The only condition on the perturbative force is that the components vary slowly with time. If the variation is too important, other averaged Gauss equations have to be used. They are not mentioned here.

$$
\begin{aligned}
\frac{da}{dt} &= \frac{2a\sqrt{1-e^2}}{\sqrt{\frac{\mu}{a}}}T \\
\frac{de}{dt} &= -\frac{3}{2}\frac{\sqrt{1-e^2}}{\sqrt{\frac{\mu}{a}}}eT \\
\frac{di}{dt} &= -\frac{3}{2}\frac{1}{\sqrt{\frac{\mu}{a}}\sqrt{1-e^2}}e\cos\omega N \\
\frac{d\Omega}{dt} &= -\frac{3}{2}\frac{1}{\sqrt{\frac{\mu}{a}}\sqrt{1-e^2}\sin i}e\sin\omega N \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{\sqrt{\frac{\mu}{a}}}R+\frac{3}{2}\frac{\cos i}{\sqrt{\frac{\mu}{a}}\sqrt{1-e^2}\sin i}e\sin\omega N \\
\frac{dM}{dt}-n &= -\frac{3}{\sqrt{\frac{\mu}{a}}}R
\end{aligned}
$$

Fig2. Averaged Gauss equation expressed in classical orbital elements

$$
\begin{aligned}
\frac{da}{dt} &= \frac{2a\sqrt{1-C^2-S^2}}{\sqrt{\frac{\mu}{a}}}T \\
\frac{dC}{dt} &= -\frac{S\sqrt{1-C^2-S^2}}{\sqrt{\frac{\mu}{a}}}R + \frac{1}{\sqrt{\frac{\mu}{a}}}\left(-C\sqrt{1-C^2-S^2} - \frac{C}{2} + \frac{C(C^2+S^2)}{2(1+\sqrt{1-C^2-S^2})}\right)T \\
&\quad -\frac{3}{2}S^2\frac{\cos i}{\sqrt{\frac{\mu}{a}\sqrt{1-C^2-S^2}\sin i}}N \\
\frac{di}{dt} &= -\frac{3}{2}\frac{C}{\sqrt{\frac{\mu}{a}}\sqrt{1-C^2-S^2}}N \\
\frac{d\Omega}{dt} &= -\frac{3}{2}\frac{S}{\sqrt{\frac{\mu}{a}}\sqrt{1-C^2-S^2}\sin i}N \\
\frac{dS}{dt} &= \frac{C\sqrt{1-C^2-S^2}}{\sqrt{\frac{\mu}{a}}}R + \frac{1}{\sqrt{\frac{\mu}{a}}}\left(-S\sqrt{1-C^2-S^2} - \frac{S}{2} + \frac{S(C^2+S^2)}{2(1+\sqrt{1-C^2-S^2})}\right)T \\
&\quad +\frac{3}{2}CS\frac{\cos i}{\sqrt{\frac{\mu}{a}}\sqrt{1-C^2-S^2}\sin i}N \\
\frac{d\lambda}{dt} - n &= \frac{1}{\sqrt{\frac{\mu}{a}}}(-3+\sqrt{1-C^2-S^2})R + \frac{3}{2}S\frac{\cos i}{\sqrt{\frac{\mu}{a}}\sqrt{1-C^2-S^2}\sin i}N
\end{aligned}
$$

Fig. 3. Averaged Gauss equations expressed in non singular elements for eccentricity

Averaged non gravitational forces have been calculated, in a numerical way, for the following forces :

1. Atmospheric drag
2. Relativistic effects
3. Radiation pressure
4. ...

3. EXAMPLES: ACCURACY REACHED, AND COMPUTATION TIME

3.1. A low altitude orbit: CHAMP

Figures 4. and 5. are the results of a numerical integration of orbital parameters of CHAMP, over 5 years. Only gravitational forces ($J_2 -> J_{15}$ field, terrestrial tides) have been considered, and the initial value has been removed to focus on the variations.

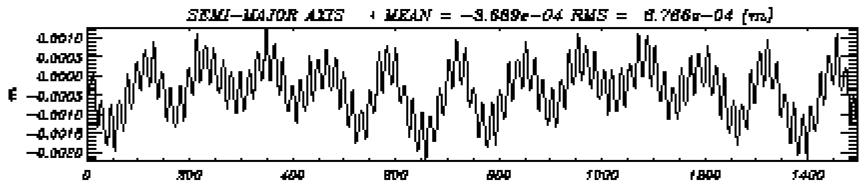


Fig. 4. Averaged motion of CHAMP: semi-major axis

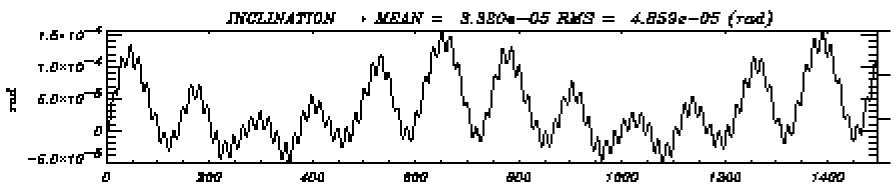


Fig. 4. Averaged motion of CHAMP: inclination

Moreover, this analytical approach affords to show the influence a particular geodynamical parameter. The following table shows the variations of e and i due to J_{15}.

Variable	Variation
e	38m
i	1cm

3.2. An example of mission analysis: GALILEO

First tests have been realized with this method to map zones of resonance for the future constellation GALILEO. Each characteristic period (a week, two weeks ...) gives a specific graphic, graphic which depends on the instantaneous coordinates of the satellites.

3.3. Time of calculation.

From a numerical point of view, this approach affords to use an integration step 100 to 1000 times higher than the integration step used in merely numerical methods: 1 hour versus 15 seconds for low satellite orbits.

The example shown below illustrates the difference of time calculation between a mere numerical integration (extrapolation), and a semi-analytical one, referring to our method. The two orbits have been computed over 5 years, with a quite complete gravitational model, and grant the same level of accuracy.

	Numerical Integration		Semi-analytical Integration	
	Integration Step	Time of calculation	Integration Step	Time of calculation
CHAMP	15s	6500s	1h	318s
GALILEO	60s	930s	12h	46s

CONCLUSION

The concept of centered motion is well adapted to the analysis of spatial missions, like those lead in CNES, in the Department of « Mathématiques Spatiales ». See [Deleflie, 2000]. This concept, already developed in classical orbital elements since ten years, is now also adapted to near circular orbits, and in particular for all low altitude satellites.

When concentrating on the only sources of long period effects, we can manage orbits over large time scales, in terms of classical orbital elements or in terms of non singular elements for eccentricity.

At last but not least, we have to mention that we are able to compare the computed centered motion -orbit- with corresponding tracking observations to be reduced: orbits deduced from tracking data and modeled orbits are linked through an orbits' filtering approach. In the context of perturbation formulations, all short period terms in « observed » orbits are removed from the osculating motion: a first step formulates short period terms on the basis of analytical theories, a second step uses a numerical filter.

As a result, our method is based on a numerical integration of a differential system where all short period effects have been removed in an analytical way. The main advantage of this method, compared with other analytic methods, is based on the fact that all perturbative forces suffered by a satellite can be taken into account. Moreover, a large integration step ensures the numerical integration errors to be much less important than in mere numerical integration. The new formulation in non singular elements for eccentricity is very important because numerous satellites have near circular orbits.

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