THE DOUBLE LUNAR SWINGBY OF THE MMS MISSION[1]

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ABSTRACT – *The double-lunar swingby (DLS) of the Magnetospheric MultiScale (MMS) mission is required, within a tight delta-V budget of 90 m/s, to change significantly the orbital elements of an initial orbit: to increase dramatically its inclination (by ≈ 57^0), decrease significantly its eccentricity (from 0.96 to 0.66) and keep its semimajor axis approximately constant. We obtain double-lunar swingbys that accomplish this task with a remarkably low delta-V of 50.3 m/s. Our approach is semi-analytical: a derived analytical expression is used to determine that a non-symmetrical targeting scheme is best suited for this problem.*

KEYWORDS: MMS, double-lunar swingby, DLS, non-symmetrical targeting, gravity assists, scattering parameters.

INTRODUCTION

The Magnetospheric MultiScale (MMS) mission is scheduled to be launched sometime in the summer of 2008. The mission will consist of four spin-stabilized spacecraft designed to fly in a tetrahedral formation. The scientific purpose of the mission is to give a better understanding of the plasma processes occurring in the Earth's magnetosphere by using the formation to obtain differential measurements of the various particles and fields of interest (see [1,2] for more details). Central to this concept being successful is a baseline mission design that can ensure passage through desired regions of the magnetosphere while simultaneously observing the requirements concerning spacecraft delta-V budget, health and safety. The baseline mission is defined to be the orbit about which the formation will fly regardless of the distribution of the individual spacecraft. The current MMS mission design calls for a baseline comprised of 4 distinct mission phases. Three of these phases (1,2, and 4) are highly elliptical orbital states. This paper focuses on phase 3: a double-lunar swingby (DLS) that transfers phase 2 to the final orbit state (phase 4).

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In the next section, we discuss all the phases of the mission but focus primarily on phase 3 (i.e. the DLS portion). To guide us in the design of the DLS, we derive an analytical expression that determines the final semimajor axis after a lunar swingby as a function of three parameters measured at periselene (referred to as scattering parameters) and the initial semimajor axis. Armed with this expression, we reason that a "non-symmetrical targeting scheme" is best suited for meeting the tentative goals of the MMS mission. The non-symmetrical targeting scheme is defined and discussed in detail in the section "Applying the Analytical Expression: Non-Symmetrical Targeting". Our numerical results show that our approach leads to acceptable solutions.

PRELIMINARY TARGETING GOALS OF MMS

The MMS mission consists of 4 distinct phases, taking the spacecraft from the low latitude inner magnetosphere, through the middle magnetosphere, into the deep magnetotail and finally into the high latitude magnetosphere. Each phase is punctuated by maneuvers that transition the spacecraft from one phase to the subsequent one. In addition, during all phases, the 4 MMS spacecraft fly in formation forming a tetrahedral structure at apogee (where applicable). For notational clarity, we take as Phase 0 the orbit the spacecraft occupy upon separation from the launch vehicle. This orbit has a perigee and apogee radius of 1.2 and 12 Earth radii (R_e), respectively, an equatorial inclination of approximately 28.5 degrees, and an argument of perigee of zero. The right-ascension of the ascending node is picked so that the desired Sun-Earth-spacecraft geometry is established [3]. The inclination of this initial orbit is immediately lowered from approximately 28.5 to 10 degrees, marking the beginning of Phase 1. Phase 1 persists for approximately 12 months, sampling low-latitude magnetospheric regions in both the sun- and tail-ward directions. At the end of Phase 1, a series of apogee-raising maneuvers are performed to boost the apogee from 12 R_e to 30 R_e. The Phase 2 orbit apogee lies entirely tailward, moving from dawn to dusk, and lasting 110 days. Throughout Phases 1 and 2, perigee maintenance maneuvers are needed to counteract the lunar perturbations that drive the radius of perigee down. At the end of Phase 2, another apogee raising maneuver is performed which raises the apogee distance to lunar distance, approximately 62 R_e. This marks the beginning of Phase 3 i.e. the DLS.

The tentative goal of the DLS of MMS is to transfer the end of the phase 2 orbital state with perigee 1.2 R_e, apogee 62 R_e and inclination ~10-20^0 (with respect to the equatorial plane) to the phase 4 orbital state with perigee 10 R_e , apogee 50 R_e and inclination 90^0 (with respect to the ecliptic plane). The delta-V allocation for phase 3 is ≈ 90 m/s. This is much less than the amount of delta-V required to accomplish the dramatic change in inclination using maneuvers. *Therefore, the DLS must accomplish most of the inclination change*. The DLS also rotates the line of apsides. The orbital states for each phase are summarized in Table 1. Note that phase 3 and phase 4 have approximately the same semimajor axis (they differ by 5%) whereas their inclination and eccentricity differ significantly.

Table 1. Orbital elements for each phase. Inclinations denoted by a $^{(1)}$ or a $^{(2)}$ are measured with respect to the equatorial and ecliptic planes respectively.

Mission Phase	Perigee (R_e)	Apogee (R_e)	Semimajor Axis (km)	Eccentricity	Inclination (deg)
Phase 0	1.2	12	42094	0.818	28.5$^{(1)}$
Phase 1	1.2	12	42094	0.818	10$^{(1)}$
Phase 2	1.2	30	99496	0.923	NA
Beginning of Phase 3 (before DLS)	1.2	~62	201544	0.962	~10-20$^{(1)}$
Phase 4	10	50	191340	0.666	90$^{(2)}$

GENERAL DESCRIPTION OF A DOUBLE LUNAR SWINGBY (DLS)

In this section, we describe briefly a typical DLS where we define our terminology in preparation for our discussion on the DLS of MMS. For more details on double-lunar swingbys we refer the reader to the following papers and references therein [4,5,6,7,8,9]. The trajectory of a DLS is depicted in Figure 1 (the normal to the lunar orbit plane points out of the page). The spacecraft in an initial orbit with semimajor axis a_i approaches the Moon, undergoes the first lunar swingby, moves in an outer loop with semimajor axis a_{out}, re-encounters the Moon for the second lunar swingby and terminates with a final semimajor axis a_f (the initial and final orbits actually have apogees that extend beyond the lunar orbit but they are drawn in figure 1 with smaller apogees to avoid cluttering the diagram). The time between lunar encounters is such that the Moon undergoes a minimum of one period. This sequence of events rotates the line-of-apsides, one of the well-known and useful properties of a DLS [4].

The first and second lunar swingby (or flyby) are qualitatively different. Let $\mathbf{V}_m$ be the linear velocity of the moon (with respect to the Earth), $\mathbf{r}_p$ be the radius vector from Moon to periselene and γ be the angle between $\mathbf{r}_p$ and $\mathbf{V}_m$ ($\mathbf{r}_p$ is not necessarily restricted to the lunar orbit plane). At the first flyby, the moon is moving *away* from the spacecraft and $90^0 < \gamma \leq 180^0$. This is called a trailing edge flyby and the spacecraft gains energy (i.e. the semimajor axis increases after the flyby). At the second flyby, the moon moves *toward* the spacecraft and $0^0 \leq \gamma < 90^0$. This is called a leading edge flyby and the spacecraft loses energy (i.e. the semimajor axis decreases).

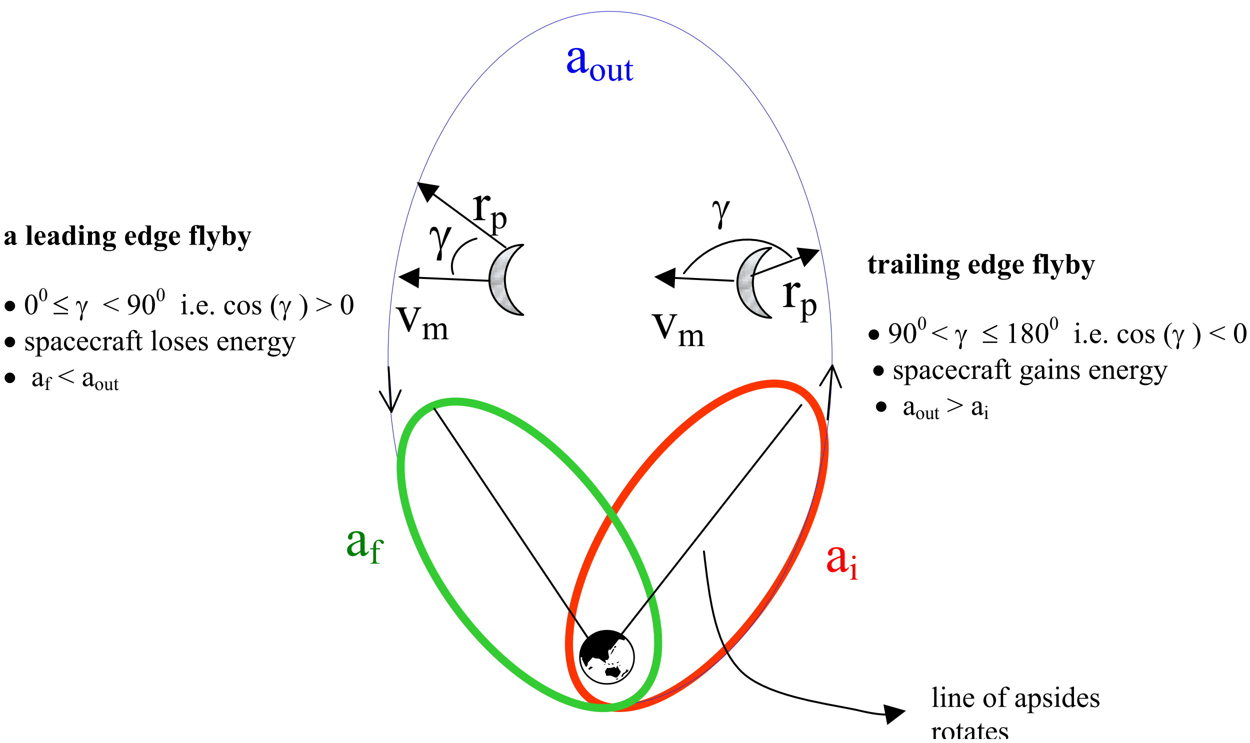

Fig. 1. Double Lunar Swingby

ANALYTICAL EXPRESSION FOR SEMIMAJOR AXIS AFTER A LUNAR SWINGBY

We derived a simple equation that determines the semimajor axis a_2 after a flyby in terms of three scattering parameters r_p, γ and V_∞ and the semimajor axis a_1 before the flyby. The equation is (see [10] for derivation)

$$\frac{1}{a_2} = \frac{1}{a_1} + \frac{4\, V_m\, V_\infty \cos(\gamma)}{\mu_E\,(1 + r_p V_\infty^2/\mu_m)} \qquad (1)$$

where

a_1	=	Earth-centered semimajor axis before flyby
a_2	=	Earth-centered semimajor axis after flyby
V_∞	=	asymptotic speed of spacecraft *with respect to the Moon*
γ	=	angle between the vector $\mathbf{v}_m$ and the vector $\mathbf{r}_p$.
r_p	=	magnitude of $\mathbf{r}_p$
V_m	=	Moon's linear speed as measured by the Earth (assumed constant)
μ_E	=	Earth's gravitational constant
μ_m	=	Moon's gravitational constant

The above equation (1) was derived in the zero sphere-of-influence approximation where the delta-V imparted by the Moon on the vehicle occurs instantaneously at one radius, namely the periselene. It also assumes the Moon to be moving in a circular orbit where the linear velocity of the Moon $\mathbf{V}_m$ is perpendicular to the Earth-Moon radius $\mathbf{r}_m$ and where the magnitude of $\mathbf{V}_m$ is constant. Equation (1) gives good quantitative results (often within 5% of the full-integrated numerical results) but its main value lies in describing how a_2 depends on the different scattering parameters: something useful as a guide for mission design. Equation (1) predicts correctly the main features of a trailing edge and leading edge flyby. For a trailing edge flyby, $90^0 < \gamma \le 180^0$, $\cos(\gamma)$ is negative and Eq.(1) predicts that $a_2 > a_1$ i.e. it predicts correctly that the semimajor axis increases and that the spacecraft gains energy. Note that for a trailing edge flyby, Eq.(1) predicts that the semimajor axis a_2 increases if the radius r_p at periselene decreases i.e. the closer to the Moon the more energy is gained. For a leading edge flyby, $0^0 \le \gamma < 90^0$, $\cos(\gamma)$ is positive and Eq.(1) predicts that $a_2 < a_1$ i.e. it predicts correctly that the semimajor axis decreases and that the spacecraft loses energy. Note that for a leading edge flyby, Eq.(1) predicts that the semimajor axis a_2 decreases if the radius r_p at periselene decreases i.e. the closer to the Moon the more energy is lost.

APPLYING THE ANALYTICAL EXPRESSION: NON-SYMMETRICAL TARGETING

The current MMS mission design requires that the final semimajor axis a_f after the DLS be 95% of the initial semimajor axis a_i before the DLS i.e. that a_i and a_f shown in figure 1 are approximately equal. This requires that the increase in semimajor axis at the first flyby be accompanied by a decrease at the second flyby of equal magnitude. The term in Eq.(1) responsible for changing the semimajor axis is $Q \equiv 4V_mV_\infty \cos(\gamma)/\mu_E(1 + r_p V_\infty^2/\mu_m)$. The sign of Q is determined by the sign of $\cos(\gamma)$. By inspecting Fig. 1, it is easy to see that $\cos(\gamma)$ is negative at the first flyby and positive at the second. If the magnitude of Q is equal at the first and second flyby, the positive and negative contributions at both flybys sum up to zero and a_i and

a_f will be approximately equal. The parameter V_∞ is almost equal at the first and second flyby (unless of course a maneuver is inserted between the two flybys). There are therefore two possible scenarios that can ensure that Q has the same magnitude at the first and second flyby. Clearly, one possible scenario is a symmetrical targeting scheme where the radius r_p and $|\cos(\gamma)|$ at the first flyby is approximately equal to the radius r_p and $|\cos(\gamma)|$ at the second flyby. However, a second scenario exists: a non-symmetrical targeting scheme where r_p and $|\cos(\gamma)|$ at the second flyby are both smaller than at the first flyby (i.e. the magnitude of Q can remain constant if both r_p and $|\cos(\gamma)|$ are decreased).

Which targeting scheme will accomplish MMS goals better: the symmetrical or non-symmetrical one? A symmetrical targeting scheme will produce a final orbit with roughly the same shape as the original (i.e. the semimajor axis a and eccentricity e will not change considerably). However, MMS requires a significant change in the eccentricity e and inclination i after the DLS. Therefore, a non-symmetrical targeting scheme is required. The dramatic increase in inclination requires the Moon to impart a large delta-V at the second flyby: much greater than the delta-V imparted by the Moon at the first flyby. Therefore, r_p at the second flyby must be much smaller than r_p at the first flyby. The procedure is therefore to target r_p and γ at the second flyby constrained by the condition that the values of r_p and $|\cos(\gamma)|$ are smaller than those of the first flyby. These constraints allow one to change the final eccentricity e and final inclination i while ensuring that the final semimajor axis is roughly equal to the initial one. *The crucial point is that r_p and $|\cos(\gamma)|$ should be significantly smaller at the second than at the first flyby*.

If one is interested only in a DLS that rotates the line of apsides, a symmetrical targeting scheme works fine. However, the DLS of MMS accomplishes something beyond the typical rotation of the line of apsides. It produces a dramatic increase in inclination i and a significant decrease in eccentricity e while simultaneously keeping the initial and final semimajor axis roughly equal. A non-symmetrical targeting scheme is therefore needed.

AN ABSOLUTE LOWER BOUND ON THE DELTA V

When the spacecraft emerge from the second lunar swingby, the MMS goal is to obtain a final orbit with perigee10 R_e, apogee 50 R_e and inclination 90^0 (with respect to the ecliptic plane). The Earth-Moon distance ranges from 56.9 R_e to 63.6 R_e so that the apogee can never be less than $\approx$ 57 R_e when the satellite emerges from the second lunar swingby. A standard burn at perigee along the velocity direction requires a Delta V of $\approx$ *33.5 m/s* to change the apogee from 57 R_e to 50 R_e. The value of 33.5 m/s can be regarded as an absolute lower bound. In other words, if everything else is perfect and the satellite emerges from the second flyby with a perigee of 10 R_e, an apogee of 57 R_e and an inclination of 90^0 there would still be a minimum requirement of 33.5 m/s to obtain the desired final state.

THE DLS OF MMS: PROCEDURE AND RESULTS

We sketch here the procedure for obtaining the DLS of MMS. The software used for our analysis includes the targeting tools *Swingby* [11] and *STK/Astrogator* [12]. The force model used for the propagators includes the point mass of the Sun and Jupiter, the gravity field of the Earth (Degree 21, Order 21) and the gravity field of the Moon (Degree 2, Order 0). The Moon is taken to be the central body in the vicinity of the first and second lunar swingby and the Earth is taken to be the central body everywhere else. The numerical integrator used is the standard RKV8(9). The positions and velocities of all planets are obtained via the DE405 Ephemeris file.

Starting with an initial state, we construct phasing loops in preparation for the first flyby. A maneuver at apogee is performed to raise perigee. This is followed by a maneuver at perigee, which controls the energy at the first flyby (i.e. it controls the scattering parameter V_∞). The semimajor axis of the outer loop is then controlled by the scattering parameters r_p and γ at the first flyby (e.g. decreasing r_p increases the outer loop semimajor axis). These two parameters are targeted so that the outer loop trajectory re-encounters the Moon in preparation for the second flyby.

The crucial element in the DLS of MMS is the targeting at the second lunar flyby. Here the guiding principle is the non-symmetrical targeting scheme discussed previously. One targets the second lunar flyby with a smaller r_p and smaller $|\cos(\gamma)|$ than at the first flyby until the best final state is reached. By targeting in this fashion, one avoids performing a blind and tedious scan of the entire parameter space. For both flybys, the scattering parameters r_p and γ are targeted by varying the argument of perigee ω and RAAN Ω of the initial state.

Detailed Chronology of Events and Numerical Results

We record in chronological sequence the numerical results of all pertinent events related to the DLS we obtained (orbit states reached, maneuvers performed, scattering parameters, etc.). Figure 2 is a realistic view of our DLS as it is taken directly from the STK-VO 3D-view after running our DLS in *STK/Astrogator*. It is possible to see the dramatic increase in inclination after the second flyby.

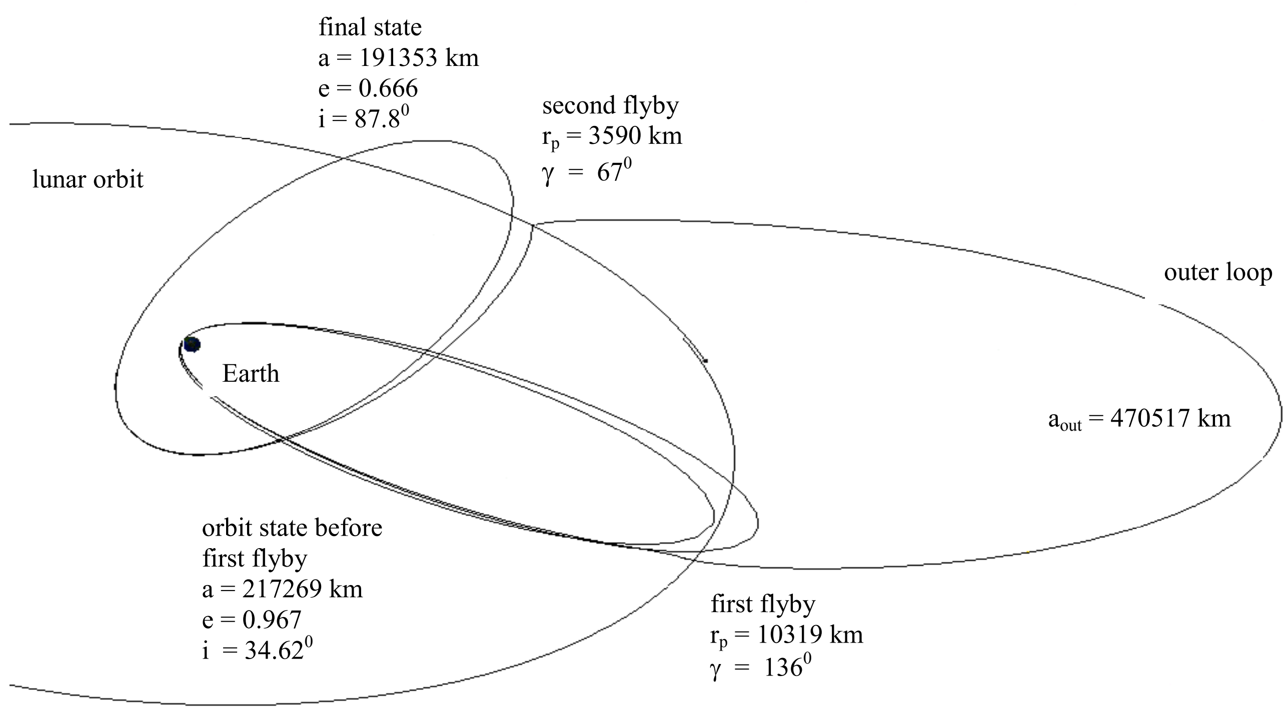

Fig.2 Realistic View of DLS

Events and Results

• *Initial state* (Epoch: March 1, 2005 04:09:27)

a = 201544 km; e = 0.962 (i.e. perigee = 1.2 Re; apogee = 62 Re)

$i = 33^0$ (Ecliptic plane); $i = 10^0$ (Equatorial plane);

- *Maneuvers performed during the phasing loops (before the first lunar flyby)*

  A 5.5 m/s apogee burn (to raise perigee) and a 6 m/s burn at perigee to reach the desired periselene at the first flyby (i.e. 11.5 m/s is used before the first flyby; both maneuvers executed along the velocity direction).

- *Orbital state* (call it "OS1") *before the first flyby (measured at perigee)*

  a = 217269 km; e = 0.967 (i.e. perigee = 1.12 Re, apogee = 67.0 Re)

  i = 34.62^0 *(ecliptic plane) ;* i = 11.3^0 *(equatorial plane)*

- *Parameters at the first flyby* (Epoch: March 25, 2005 22:21:43)

  r_p = 10319.44 km ; γ = 136 deg ($|\cos(\gamma)|$ = 0.72) ; V_∞ = 0.982 km/s

- *Orbital state at apogee of outer-loop*

  a = 470517 km ; e = 0.864

  i = 5.02^0 (ecliptic plane) ; i = 27.82^0 (equatorial plane)

- *Parameters at the second flyby* (Epoch: April 27, 2005 23:45:23)

  r_p = 3590 km ; γ = 67 deg ($\cos(\gamma)$ = 0.39) ; V_∞ = 1.13 km/s

- *Orbital state*(call it "OS2") *reached at the first perigee immediately following the second flyby*

  a = 215041 km; e = 0.705 (i.e. perigee = 9.95 Re, apogee = 57.49 Re)

  i = 86.57^0 *(ecliptic plane) ;* i = 89.89^0 *(equatorial plane)*

* The best orbital state one could have achieved after the second lunar flyby is a 10 x 57 Re with an inclination of 90 deg with respect to the ecliptic. So we are very close to the best possible achievable state coming out of the second lunar flyby.

- *Maneuvers performed at first perigee and first apogee following second flyby*

  Delta V at first perigee following second flyby: - 34.4 m/s

  Delta V at first apogee following second flyby : + 4 m/s

  (both maneuvers executed along the velocity direction)

  Delta V used for the perigee and apogee burn after the second flyby is 38.8 m/s. This is very close to the absolute lower bound of 33.5 m/s (discussed previously in section on "Lower Bound").

- *Final state reached (measured at second perigee following second flyby)*

  a = 191353 km ; *e* = 0.667 (i.e. perigee= 9.99 Re ; apogee = 50.01 Re)

  i = 87.8 deg (with respect to the ecliptic plane), *i* = 91.00 deg (with respect to the equatorial plane),

  Total Delta V = 11.5 + 38.8 = *50.3 m/s*

ANALYSIS OF RESULTS

We now analyze the results obtained in the previous sub-section. The final state of the MMS mission is required to have a perigee of 10.0 Re, apogee of 50.0 Re and an inclination of 90^0 with respect to the ecliptic plane. We obtained a final state with a perigee of 9.99 Re, an apogee of 50.01 Re and an inclination of 87.8^0 with respect to the ecliptic plane (91^0 with respect to the equatorial plane). *The delta-V used to transfer the initial state to the final state is 50.3 m/s.* This is considerably less than the 90 m/s delta-V allocation. *We have therefore met the baseline requirements for the DLS portion of MMS.*

The delta-V of 50.3 m/s is the sum of the delta-V before the first flyby (11.5 m/s) and the delta-V after the second flyby (38.8 m/s). The value of 38.8 m/s is remarkably close to the absolute lower bound of 33.5 m/s discussed previously. We had shown in a previous section that one requires a minimum delta-V of 33.5 m/s to achieve the desired final state after the second flyby. In other words, the best state one can hope to achieve after the second flyby is one that still requires a delta-V of 33.5 m/s. We required 38.8 m/s to reach the desired final state. This implies that the state we obtained at perigee following the second flyby is close to being the ideal one (where the ideal one requires 33.5 m/s).

Non-symmetrical Targeting Scheme

The orbital state OS1 occuring before the first flyby has a semimajor axis of 217269 km, an eccentricity of 0.967 and an inclination of 34.62^0 with respect to the ecliptic plane (see sub-section "Events and Results"). The orbital state OS2 immediately following the second flyby has a semimajor axis of 215041 km, an eccentricity of 0.705 and an inclination with respect to the ecliptic plane of 86.57^0. The DLS is the only agent responsible for transferring OS1 to OS2 as there are no maneuvers between these two states. The DLS accomplishes by itself a 52^0 inclination change and lowers the eccentricity from 0.967 to 0.705. Note that the DLS changes significantly the inclination and eccentricity while keeping the semimajor axis constant (i.e. the semimajor axis of OS1 and OS2 are almost equal). These results were obtained by applying the non-symmetrical targeting scheme discussed in the section "Applying the Analytical Expression:Non-Symmetrical Targeting". We argued in that section that r_p and $|\cos(\gamma)|$ should be significantly smaller at the second than at the first flyby. We also argued that the quantity $Q \equiv 4V_mV_\infty \cos(\gamma)/\mu_E(1 + r_p V_\infty^2/\mu_m)$ has the same magnitude at both flybys if the semimajor axis before and after the DLS (i.e. at OS1 and OS2) is close to being equal. We now have numerical data to verify these claims.

At the second flyby the scattering parameters are r_p = 3590 km, γ = 67 deg ($\cos(\gamma)$ = 0.39) and V_∞ = 1.13 km/s while at the first flyby they are r_p = 10319 km, γ = 136 deg ($|\cos(\gamma)|$ = 0.72) and V_∞ = 0.982. Clearly, r_p and $|\cos(\gamma)|$ are significantly smaller at the second than at the first flyby in accord with our non-symmetrical targeting scheme. Let us now compare the value of Q at the first and second flyby. The equation for Q assumes that the linear speed of the Moon V_m is constant (= 1.018 km/s). Substituting r_p, γ, and V_∞ into Q we obtain Q = 2.386 x 10^{-6} km^{-1} at the first flyby and Q = 2.3266 x 10^{-6} km^{-1} at the second flyby. The difference between the two values is less than 3%. Our non-symmetrical targeting scheme has therefore proven to be very successful.

Numerical Verification of Analytical Expression

The derived Eq. (1) was used to determine that a non-symmetrical targeting scheme was the most suited for the main goals of MMS. This scheme worked extremely well and demonstrates the usefulness of Eq.(1) in designing the DLS. In this section we check to see if the analytical expression also gives good quantitative results. We calculate the semimajor axis after the first and second flybys using Eq.(1) and compare them to the numerical results quoted previously.

To calculate a_2 from Eq.(1) we need the values of the following variables: a_1, r_p, γ and V_∞. For the first flyby these are: a_1 = 217269 km ; r_p = 10321 km; $\gamma = 136^0$; V_∞ = 0.982 km/s. Substituting these values into Eq.(1) yields a_2 = 450150 km. This is to be compared to the value of the semimajor axis of the outer loop after the first flyby i.e. 470517 km. The difference between the analytical and numerical result is 4.3 %. For the second flyby one has: a_1 = 470517 km ; r_p = 3590 km; $\gamma = 67^0$; V_∞ = 1.13 km/s. Substituting these values into Eq.(1) yields a_2 = 224400 km. This is to be compared to the semimajor axis at perigee immediately following the second flyby i.e. 215041 km. The difference between the analytical and numerical result is 4.2 %.

There is therefore very good quantitative agreement between the analytical and numerical results despite the fact that the analytical expression does not take into account solar and lunar perturbations.

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